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APLANATIC WAXICONS*

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Abstract

Waxicon mirror components offer many advantages in designing optical systems for transporting high power laser beams. The widespread use of waxicons has been limited because of their high sensitivity to tilt errors. This paper gives the equations of the surfaces of a waxicon that is rigorously corrected for both spherical aberration and coma. Computer ray tracing has confirmed its low sensitivity to tilt errors: if the aplanatic waxicon as a whole is tilted by a small angle δ, the RMS wavefront error in the output beam will be proportional to δ^2.

Introduction

The waxicon mirror element is a useful component in optical design, particularly in systems for transporting high power annular laser beams. The simplest application of a waxicon is as a spiderless beam expander or beam compactor.(1) The use of waxicons has been considered for unstable resonators(2) or laser amplifiers(3) having an annular gain medium. There are many situations where the number of reflecting surfaces, the number of mirror mounts, or the overall size of the system can be significantly reduced by using waxicons. Optical components of this type have been built and experimentally tested.(1, 2) Precision machining of metal mirrors(4) now makes it possible to fabricate exotic surfaces with an accuracy acceptable at infrared and far infrared wavelengths. In practice it has been found(2) that the design advantages of waxicons are offset by the fact that their optical performance is extremely sensitive to tilt errors. This drawback has hindered the widespread application of waxicons. An analysis of the aberrations involved shows that the high tilt sensitivity is produced mainly by coma: the waxicon designs considered in the past strongly violate the Abbe sine condition. The aplanatic waxicons described in this paper are rigorously corrected for both coma and spherical aberration. Analytic formulas for the waxicon surfaces necessary to produce these properties are derived in the next section. Two examples of aplanatic waxicons, and the results of a computer ray trace are presented.

Equation of the Aplanatic Waxicon

The derivation is similar to the one given by Schwarzschild(5) for the equation of the aplanatic telescope. Schwarzschild's solution does not include waxicon configurations. An ab initio derivation is needed to obtain the desired formulas. A graphical construction of the surfaces is also possible, using a technique described by Luneburg.(6)

In Figure 1 the z-axis is the axis of symmetry and ρ is the radial distance from this axis. The ray shown enters and leaves the waxicon parallel to the z-axis; it intersects the first surface at the point (ρ_1, z_1) and the second surface at (ρ_2, z_2). The angle of incidence is denoted by β_1 at the first surface and β_2 at the second surface. The shape of the surfaces is determined by two defining conditions. The first is the conservation of path length (Fermat's principle). With the notation in Figure 1, this condition is expressed as

$$R - z_1 - z_2 = 2c \quad \text{(c constant)}. \tag{1}$$

The second condition,

$$\rho_2 = m\rho_1 \quad \text{(m constant)}, \tag{2}$$

is equivalent to the Abbe sine condition for conjugate points at infinity. It ensures that the waxicon is coma-free.

Referring to Figure 1, we write the slope of surfaces 1 and 2 as

$$\tan \alpha_1 = d\rho_1/dz_1, \tag{3}$$

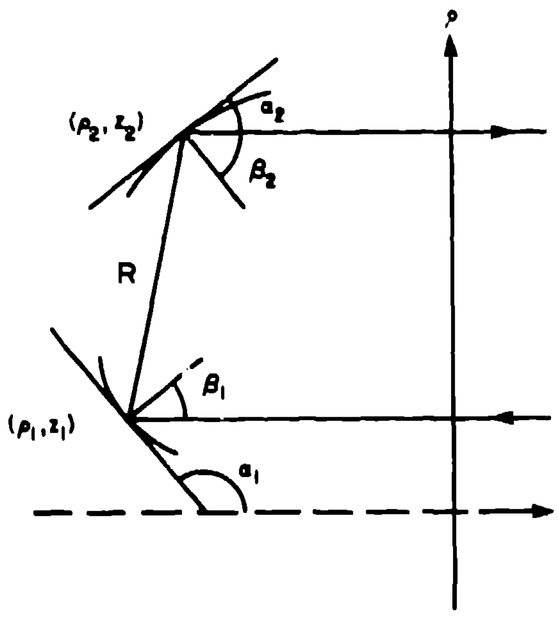

Figure 1. Notation used for waxicon geometry

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$$\tan \alpha_2 = d\rho_2/dz_2. \tag{4}$$

Inspection of Figure 2 gives the following relations between the angles:

$$\beta_1 + \beta_2 = \pi/2, \tag{5}$$

$$\alpha_1 - \beta_1 = \pi/2, \tag{6}$$

$$\alpha_2 + \beta_2 = \pi/2. \tag{7}$$

By examining the relation between (ρ_1, z_1) and (ρ_2, z_2) one sees that

$$\tan 2\beta_1 = (\rho_2 - \rho_1)/(z_2 - z_1), \tag{8}$$

$$R^2 = (\rho_2 - \rho_1)^2 + (z_2 - z_1)^2. \tag{9}$$

If the information in Eq. (1) is used in Eq. (9), the latter simplifies to

$$(\rho_2 - \rho_1)^2 = 4(z_1 + c)(z_2 + c). \tag{10}$$

Our objective is to obtain an expression for $d\rho_1/dz_1$ which depends only on ρ_1 and z_1. From Eqs. (3) and (6) we have

$$d\rho_1/dz_1 = \tan \alpha_1 = \tan (\beta_1 + \frac{\pi}{2}) = -\cot \beta_1. \tag{11}$$

We now use the trigonometric identity

$$\cot \beta = \frac{1}{\tan 2\beta} + \frac{1}{\sin 2\beta}, \tag{12}$$

which enables us to express Eq. (11) as

$$\frac{d\rho_1}{dz_1} = -\frac{z_2 - z_1}{\rho_2 - \rho_1} - \frac{R}{\rho_2 - \rho_1}. \tag{13}$$

Equation (10) enables us to eliminate z_1 from Eq. (13). We have

$$z_2 - z_1 = -(z_1 + c) + \frac{1}{4}(\rho_2 - \rho_1)^2/(z_1 + c), \tag{14}$$

$$R = z_1 + c + \frac{1}{4}(\rho_2 - \rho_1)^2/(z_1 + c), \tag{15}$$

which gives

$$\frac{d\rho_1}{dz_1} = -\frac{1}{2}\frac{\rho_2 - \rho_1}{z_1 + c}. \tag{16}$$

Equation (2) now enables us to eliminate ρ_2:

$$\frac{d\rho_1}{dz_1} = -\frac{m - 1}{2}\frac{\rho_1}{z_1 + c}. \tag{17}$$

This differential equation is easily integrated:

$$z_1 + c = a\,\rho_1^{-2/(m-1)}, \tag{18}$$

where a is a constant of integration.

For convenience we will allow ρ to take on negative values; the point $(-\rho, z)$ is related to (ρ, z) by a rotation of π radians about the z-axis. To simplify the form of the solution we choose the origin of z at the point where $d\rho_1/dz_1 = -1$, or equivalently where $d\rho_2/dz_2 = 1$. From Eqs. (17) and (18) we have

$$c = \frac{m-1}{2} b, \tag{19}$$

$$c = a\, b^{-2/(m-1)}, \tag{20}$$

where b is the value of ρ_1 when $z_1 = 0$. The constant a is fixed by these two conditions:

$$a = c \left[\frac{m-1}{2c}\right]^{-2/(m-1)} . \tag{21}$$

The equation of surface 1 can thus be written as

$$\rho_1 = \frac{2c}{m-1} \left[1 + \frac{z_1}{c}\right]^{(1-m)/2} . \tag{22}$$

By combining Eqs. (2), (10), and (18), we can now obtain the equation of surface 2

$$z_2 + c = \frac{1}{4a} (m-1)^2 (\rho_2/m)^{(2m)/(m-1)} . \tag{23}$$

Using the value of a from Eq. (21), we find for surface 2

$$\rho_2 = \frac{2mc}{m-1} \left[1 + \frac{z_2}{c}\right]^{(m-1)/(2m)} . \tag{24}$$

The equations of the two surfaces are interchanged if we replace m by 1/m.

Mansell and Saito[1] described a waxicon which will convert an annular beam with a uniform intensity profile into one with a gaussian intensity profile (or vice versa). The aplanatic waxicons described by Eqs. (22), (24) preserve the shape of the intensity profile. Let $I_1(\rho_1)$ be the intensity distribution in the input beam, and $I_2(\rho_2)$ that in the output beam. If we neglect polarization effects, energy conservation gives

$$I_2(\rho_2)\, \rho_2\, d\rho_2 = I_1(\rho_1)\, \rho_1\, d\rho_1 . \tag{25}$$

From Eq. (2) we have

$$I_2(\rho) = m^{-2} I_1(\rho/m). \tag{26}$$

In particular, if the input beam has a uniform intensity profile, the output beam will also be uniform.

Examples of Aplanatic Waxicons

For $m = 1$ the solutions degenerate into a plane mirror $z = -c$. Another simple case occurs when $m = -1$; we obtain a right angle conical axicon described by $\rho_1 = -c - z_1$, $\rho_2 = c + z_2$. A beam expander design with $m = 5$ and $c = 0.35$ meters is shown in Figure 2. The range of ρ in meters is $0.15 \le \rho_1 \le 0.20$ for the input beam and $0.75 \le \rho_2 \le 1.0$ for the output beam. As seen in Figure 2, a diffuse ring focus is obtained at a radius of about 0.29 meters. This waxicon design was considered as an input mirror for the six annular laser amplifiers in the Antares CO_2 laser under construction at the Los Alamos Scientific Laboratory. The use of a waxicon input mirror would reduce the overall length of the amplifier, and the ring focus would serve for retropulse isolation. There are several other applications where it would be useful to produce a diffuse ring focus without affecting the optical quality of the beam: e.g. in saturable absorber cells, and in some applications in photochemistry.

Numerical ray tracing confirmed that this waxicon is insensitive to tilts, to first order in the tilt angle. The ray trace was performed on MAXWELL, an optics computer code specially designed to handle extreme aspheric surfaces.[7] We used 100 rays parallel to the z-axis, randomly distributed over the entrance annulus. If the waxicon in Figure 2 is tilted as a whole by a small angle δ, the RMS wavefront deviation will be $0.016\,\delta^2$ meters. This formula was verified for 5 valued of δ between 10^{-6} and 10^{-2} radians.

Figure 3 shows another waxicon design with $m = -5$ and $c = 0.35$ meters. The input and output annuli are the same as for the case shown in Figure 2. The waxicon in Figure 3 produces a line focus on the z-axis on an interval given by $0.06 \le z \le 0.14$ meters. This case is included to illustrate the type of solutions that occur. It is less practical than the example in Figure 2: the mirror thickness is greater, and the volume of the diffuse focus considerably smaller.

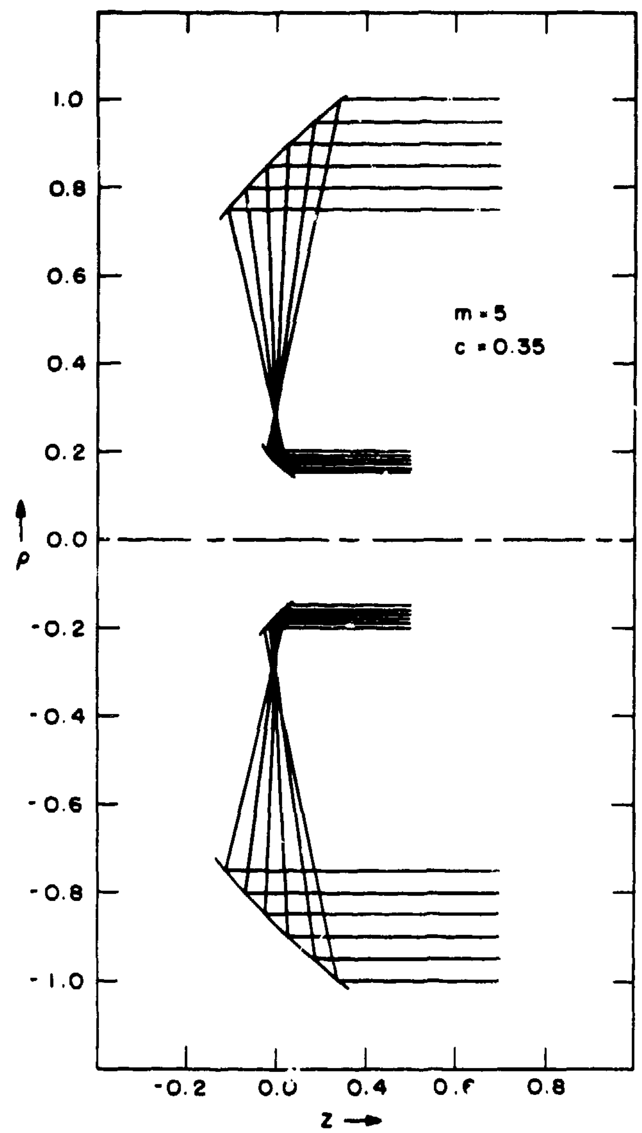

Figure 2. Cross section of an aplanatic waxicon with positive magnification

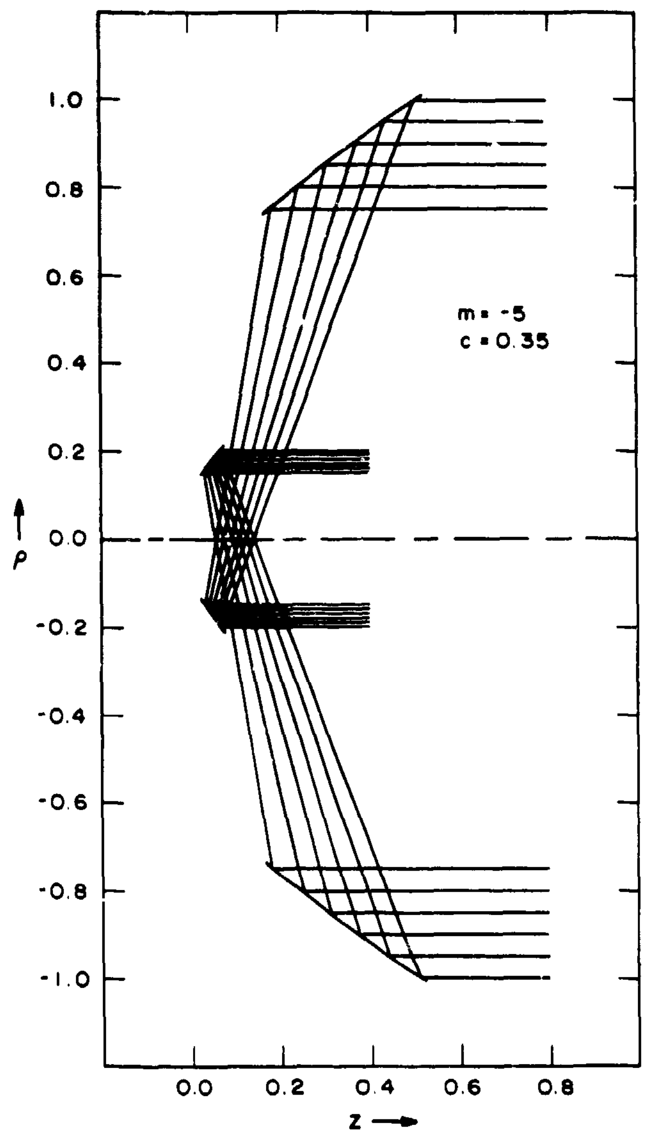

Figure 3. Cross section of an aplanatic waxicon with negative magnification

Conclusions

Waxicons with reflecting surfaces specified by Eqs. (22) and (24) are fully corrected for spherical aberration and coma. If the input beam is a uniform annular beam parallel to the z-axis, the output beam will also be uniform and parallel to the z-axis. The optical quality of the output beam will not be affected by tilting the waxicon, to first order in the tilt angle. The designs with m > 0 produce a diffuse ring focus which should be useful for several applications.

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